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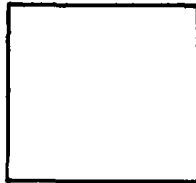
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INERTIAL GEODESY

by

H. Moritz



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## EDITED TRANSLATION

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Helmut Moritz\*

## Inertial Geodesy

### 1. Introduction

For several years now inertial apparatus mounted in a ground vehicle or a helicopter have been used to rapidly measure differences in geodesic coordinates and height with an accuracy better than a meter on routes of less than 100 kilometers [1]. It should be stated that these apparatus use inertial systems almost identical to those used in navigation instruments on B747 or Concorde aircraft. From the geodesic standpoint, aircraft navigation instruments are not accurate: on the Paris-New York run the aircraft fixing position error can exceed many kilometers. The question then is whether it is possible to obtain geodesic accuracy with such an apparatus.

### 2. Inertial Navigation Principles

Let us look at the path taken by the ground vehicle or helicopter; it

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IUGG.

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can be expressed in the Cartesian system xyz as a function of time t:

$$\begin{aligned}x &= x(t), \\y &= y(t), \\z &= z(t),\end{aligned}\tag{1}$$

or in vector form:

$$\underline{x} = \underline{x}(t).\tag{2}$$

Then speed will be a derivative of vector x relative to time:

$$\dot{\underline{x}} = \frac{d\underline{x}}{dt},\tag{3}$$

and acceleration will be a second derivative:

$$\ddot{\underline{x}} = \frac{d^2\underline{x}}{dt^2}.\tag{4}$$

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Vector x components can be observed using three accelerometers. An inertial platform with gyroscope stabilizes the coordinates axes. The inertial platform with the accelerometers forms the core of the navigation instrument.

The principle of defining coordinate differences between two points A and B is very simple: acceleration  $\ddot{\underline{x}}(t)$  registers as a function from t on the entire pathway between A and B; integration relative to t gives speed

$$\dot{\underline{x}} = \int_{t_1}^t \ddot{\underline{x}} dt\tag{5}$$

at each point of pathway AB and by repeated integration we obtain the sought difference in coordinates:

$$\underline{x}_B - \underline{x}_A = \int_{t_1}^{t_2} \dot{\underline{x}} dt;\tag{6}$$

$t_1$  and  $t_2$  correspond to points A and B. All integration is automatic and continuous during travel; as a consequence  $\dot{x}(t)$  and  $x_B - x_A$  are recorded. The solution will be unequivocal if <sup>we</sup> assume that  $\dot{x} = 0$  at point A. The assumption is obvious since we stop the ground vehicle at points A and B.

For the same reason, the following should occur:

$$x_s = \int_{t_1}^{t_2} \dot{x} dt = 0, \quad (7)$$

in other words, at point B we should read a zero speed (just as at point A). The use of geodesy is based on this simple fact.

Actually inaccuracies in inertial navigation are caused mostly by the motion of gyroscopes and accelerometers (this phenomenon is well-known in gravimeters). The motion is quasilinear. Because of motion, the speed vector at point B will not be precisely zero; this interval from zero is used to determine motion and correct the results.

Practically, this makes it necessary to stop the ground vehicle or helicopter at three to five minute intervals, record  $\dot{x}$  each time and, if necessary, set  $\dot{x} = 0$ . The success of this method is based on this.

### 3. Geodesic Applications

Almost all geodesic measurements depend on the earth's gravitational field. If inertial measurements are to be useful in geodesy, they must relate to this field. Therefore, the geodesic inertial instrument should obtain information on the gravitational field, i.e., on the force of gravity  $g$  vector. This vector is defined by its length, which is the acceleration of force of gravity  $g$ , and by its direction, which is in line with the vertical direction determined by astronomical width  $\phi$  and astronomical length  $\lambda$ .

Observation of  $g$  is not difficult: the accelerometer in the vertical position acts as a gravimeter. Information on  $\phi$  and  $\lambda$ , as on azimuth  $A$ , can

be obtained with gyrocompasses which are oriented relative to the axis of the earth's rotation and the vertical direction.

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On the other hand a close mathematical relationship exists among the Cartesian coordinates  $x, y, z$  and coordinates  $\varphi$  (geodesic width),  $\lambda$  (geodesic length and  $h$  (height over ellipsoid). If we assume that axis  $z$  is parallel to the axis of the earth's rotation and that axis  $x$  complies with Greenwich meridian, we have

$$\begin{aligned} x &= (N+h)\cos\varphi\cos\lambda, \\ y &= (N+h)\cos\varphi\sin\lambda, \\ z &= (b^2N/a^2+h)\sin\varphi, \end{aligned} \quad (8)$$

where  $a$  and  $b$  designate ellipsoid semi-axes, and  $N$  is a known function from  $\varphi$  [6], page 182.

The geodesic inertial instrument, therefore, is made up of two fundamental parts:

1. Navigation system, by which Cartesian coordinates  $x, y, z$  are obtained, or geodesic  $\varphi, \lambda, h$  (these two systems are synonymous in view of (8)),

2. Gyrocompass system to determine azimuth  $A$  and astronomical coordinates  $\Phi$  and  $\lambda$  ( $\Phi$  and  $\lambda$  define the vertical direction and are totally different from  $\varphi$  and  $\lambda$ ).

The six following quantities can be determined by such a geodesic instrument:

--three coordinates,

--two components  $\xi, \eta$  vertical deflection, defined by

$$\xi = \Phi - \varphi \quad \text{and} \quad \eta = (\lambda - \lambda)\cos\varphi,$$

--acceleration  $g$ .



Strictly speaking, we do not obtain the absolute values of these quantities; we obtain their differences between two neighboring points.

The method of determining these quantities depends on the construction of the inertial platform. Three basic types of such platforms are:

Type I: Cartesian platform. Platform axes maintain their directions, always remaining parallel to their exit positions.

Type II: Ellipsoidal platform. Platform axes change directions so that axis  $z$  is always perpendicular to the reference ellipsoid, and axis  $x$  is tangent to the ellipsoid meridian.

Type III. Local platform. Platform axes change direction so that at every point of the route, axis  $z$  is in line with the local vertical line, i.e., with the direction of vector  $g$ , and axis  $x$  has an astronomical azimuth  $A = 0$ .

Type I is the simplest case: axis direction is always the same (at least under ideal conditions), the same Cartesian system  $xyz$  remains throughout. This is why we used this system in the previous section to explain the principles of inertial navigation. Instead of differences  $\Delta x$ ,  $\Delta y$ ,  $\Delta z$ , coordinate differences  $\Delta\varphi$ ,  $\Delta\lambda$ ,  $\Delta h$  can be obtained easily if the instrument contains a computer which converts coordinates on the basis of model (8).

In Type II such conversions are not only made mathematically but they also produce a constant change in axis direction by the above-mentioned method. Due to the analytical relationship (8) between systems  $xyz$  and  $\varphi\lambda h$ , such a change can be made "internally" without external information on the true vertical direction defined by  $\Phi$  and  $A$ ; gyrocompasses have no effect on axis orientation.

From this viewpoint, Types I and II are similar: In Type I, conversions (8) are made numerically, i.e., digitally, and in Type II, calculations (8) are made physico-mechanically, i.e., by analog.

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In both types we obtain  $\varphi$ ,  $\lambda$ ,  $h$ ; juxtapositioning  $\varphi$ ,  $\lambda$ , with  $\Phi$ ,  $\Lambda$ , observed by gyrocompasses, we obtain components  $\xi$ ,  $\eta$  of vertical deflection (and accurate differences  $\Delta\xi$  and  $\Delta\eta$  between two stations).

Type III differs greatly from the previous two by the fact that the true local vertical direction and astronomical azimuth have a direct influence on axis orientation, accomplished with gyrocompasses.

In actual practice, axis fixing is done discretely, only at the points at which the ground vehicle or helicopter is brought to a stop; between these stations axis rotation complies with the ellipsoidal model, exactly as in Type II. Despite this, the difference between Types II and III is significant for there is no direct mathematical relationship between  $\Phi$ ,  $\Lambda$  and xyz in the sense (8), due to the irregularity of the earth's gravitational field.

Determination of vertical deflection by Type III apparatus is based on the fact that between stations the apparatus functions in accordance with the ellipsoidal model, and so the quantities of changes in axis fixing at the stations express deflection of true vertical from vertical ellipsoidal (when the apparatus is functioning perfectly).

How significant are coordinates obtained by Type III apparatus? In its internal local system XYZ, which is constantly changing, dZ denotes exactly the same thing as an increase in height observed in levelling. By integrating dZ we obtain with sufficient accuracy (omitting orthometric corrections) the difference  $\Delta H$  of the height of orthometric H, i.e., height above geoid.

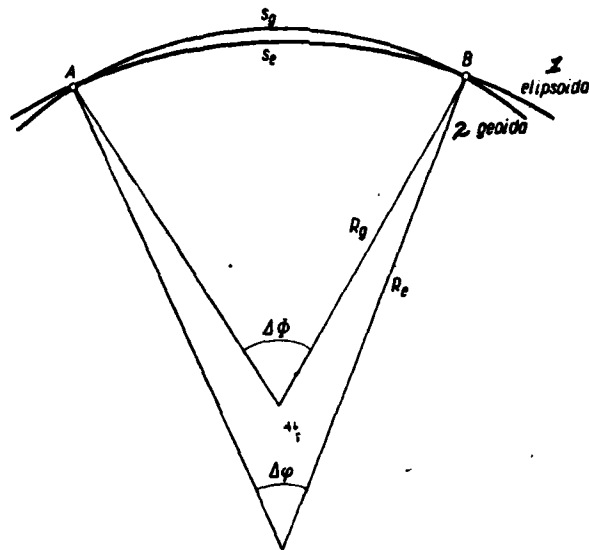


Fig. 1

Key: 1. Ellipsoid  
2. Geoid

Similar increases in  $dX$  and  $dY$  give, after integration, difference  $\Delta X$  and  $\Delta Y$  between two stations. Components  $\Delta X$  and  $\Delta Y$  are measured on the geoid surface; they are at approximation distances on the geoid. The instrument computer converts these components to geodesic coordinates  $\Delta\varphi$  and  $\Delta\lambda$  on the basis of a mathematical model of a reference ellipsoid.

Grafarend's [3] theory of "anholonomic systems" gives us an exact interpretation of  $dX$ ,  $dY$ ,  $dZ$  integration. Within the limits of measurement accuracy it can be stated that Type III apparatuses measure  $\Delta\varphi$ ,  $\Delta\lambda$  and  $\Delta H$ . It should be noted that  $\Delta\varphi$  and  $\Delta\lambda$  are referred to, and not  $\Delta\phi$  and  $\Delta\lambda$ .

This situation can be explained using the simple case shown in Fig. 1. Let us look at two points A and B lying in the same meridian, which are intersected by the geoid and ellipsoid. Assuming that the geoid and ellipsoid arcs between A and B can be approximated through circular arcs with  $\underline{R_g}$  or  $\underline{R_e}$  rays, then arc  $\underline{S_g}$  measured by a Type III apparatus differs only very slightly from arc  $\underline{S_e}$  (measured by Type II). The instrument computer, using  $\underline{R_e}$  taken from the ellipsoid model, calculates

$$\frac{s_g}{R_e} \pm \frac{s_e}{R_e} = \Delta\varphi,$$

instead of

$$\frac{s_g}{R_e} = \Delta\varphi,$$

which is what we wanted to show.

In summary, all three types of instruments give  $\varphi$ ,  $\lambda$ , coordinate differences,  $\xi$ ,  $\eta$  vertical deflection and  $g$  acceleration. As to height: Types I and II measure the difference in height  $h$  above the ellipsoid and Type III gives the difference in height  $H$  above the geoid.

#### 4. Practical Aspects

According to [5] inertial systems can be divided into three categories: systems of low, average and high precision. Low precision systems are rated at over 2 km/hr and are not suitable for geodesic applications. Average precision systems are rated at about 1.5 km/hr; they can be used to measure horizontal coordinates and height to an accuracy of 1-3 meters,  $\Delta g$  to 2-5 mgal accuracy, and  $\xi$  and  $\eta$  to 1.5-5". High precision systems are rated at 0.2 to 0.5 km/hr at the following

accuracies;

coordinates and height	< 0.5 m
gravity force acceleration	< 2 mgal
vertical deflection	< 1.5"

Inertial measurements in a ground vehicle or helicopter are made as follows. First the instrument must be warmed up, calibrated and oriented: z and x axes are oriented to vertical and meridian at the exit point (in Type I the axes maintain these directions, while in the other types the axes will change correspondingly); this process takes about an hour. Then the ground vehicle rides (or the helicopter flies) from station to station. It is important to stop every 3-5 minutes to verify zero speed; this takes only about a minute. In this way measuring can continue all day; the ending point will be the point of known coordinates, same as the exit point.

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The first geodesic inertial system was the Auto-Surveyor made by Litton Systems and used in the last several years in the United States and Canada [4]; this is a Type III apparatus. The newest high precision instrument, in the sense described above, is Geo-Spin (Type II) made by Honeywell. Independent of these American companies, the Ferranti firm in Edinburgh is also producing a geodesic system.

Inertial systems are very useful, particularly in the developing countries and in large countries such as Canada or Australia. Inertial measurements can be used to replace or supplement classic triangulation, particularly in combination with doppler type satellite measurements. The method can be still further improved; references in literature on this subject speak of future inertial systems rated on the order of 0.3 m/hr [2]! We are at the beginning of the inertial geodesy era.

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## Summary

In the last few years, inertial measuring systems have been developed, which are installed in a ground vehicle or a helicopter and allow rapid surveys with accuracies better than a meter. The systems consist of a stable platform to define the coordinate axes, of accelerometers to measure accelerations and gravity, and of gyrocompasses to refer the measuring system to the local direction of gravity and to local north. Three-dimensional coordinate differences are found by twice integrating accelerations, and relative deflections of the vertical are also obtained. The present paper discusses physical and geodetic principles and outlines practical aspects.

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